D R A F T   R E P O R T

MODAL ANALYSIS. NATURAL FREQUENCIES AND MODE SHAPES OF 3D BEAM

All dynamic analysis types are based on the following general equation of motion for a finite element system:

$$\underset{n\times n}{[M]}\underset{n\times 1}{\{\overset{\circ\circ}{q}\}}+\underset{n\times n}{[C]}\underset{n\times 1}{\{\overset{\circ}{q}\}}+\underset{n\times n}{[K]}\underset{n\times 1}{\{q\}}=\underset{n\times 1}{\{F(t)\}}, \qquad (1)$$

where: [M] mass matrix, [C] damping matrix, [K] stiffness matrix, $\{q\}$ nodal displacement vector, $\{\dot{q}\}$ nodal velocity vector, $\{\ddot{q}\}$ nodal acceleration vector, $\{F(t)\}$ load vector, (t) time.

Modal analysis

For modal analysis, the ANSYS program assumes free (unforced) vibration with no dumping, described by the following equation of motion:

$$\underset{n\times n}{[M]}\underset{n\times 1}{\{\overset{\circ\circ}{q}\}}+\underset{n\times n}{[K]}\underset{n\times 1}{\{q\}}=\underset{n\times 1}{\{0\}} \qquad (2)$$

The equation reduces to the eigenvalue problem:

$$\left([K]-\omega^2[M]\right)\{q\}=\{0\} \qquad (3)$$

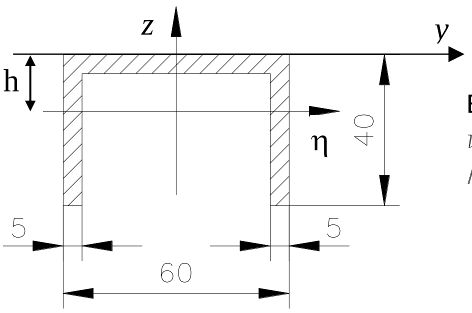
We are interested in non-trivial solutions that meet the condition:

$$\det\left([K]-\omega^2[M]\right)=0 \qquad (4)$$

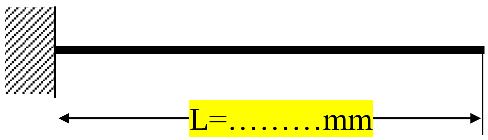
The above condition provides the natural frequencies ω_i. Each natural frequency is associated with the eigenvector $\{q\}_i$ describing the shape of the deformation at the free vibration with the frequency ω_i (mode shape). The smallest natural frequency is called fundamental frequency of vibration.
The mode shape is defined by relations between DOF – the magnitudes of the nodal displacements have no meaning. The eigenvector may be arbitrary scaled - it is usually normalized in relation to unity matrix or to mass matrix: $\lfloor q \rfloor_i [M]\{q\}_i=1$.

P R O B L E M

Find the first 8 natural frequencies and the associated mode shapes of the 3D cantilever beam.



E=2*10^5 MPa
ν=0.3
ρ=8*10^3 $\frac{kg}{m^3}$



J=J$_\eta$=1.0097 10^{-7}m^4
A=0.6·10^{-3}m^2 ,   h=13.27mm

Fig. 1. Cross-section of the beam

The analytical solution for one-dimensional beam model (bending only):

$$\omega_1^s=3.5156\cdot\frac{1}{l^2}\sqrt{\frac{EJ}{\rho A}},$$

$$\omega_2^s=22.0346\cdot\frac{1}{l^2}\sqrt{\frac{EJ}{\rho A}}, \qquad (5)$$

$$\omega_i^s=\left[\frac{(2i-1)\pi}{2}\right]^2\cdot\frac{1}{l^2}\sqrt{\frac{EJ}{\rho A}}, \qquad i=3,4...,$$

ATTENSION on the selection of units: SI (N, m, s, kg) or mod_SI (N, mm, s, t)

D R A F T   R E P O R T

Number of elements=…………… Number of nodes=……………..

Table 1a C A S E 1 . Analyse the cantilever beam using solid elements (Solid185).

Mode	frequency **f** $_{FEM}$ [Hz]	Natural freq. ω_{FEM} [rad/s]	Shape description
1			
2			
3			
4			
5			
6			
7			
8			

Table 1b. Theoretical results for a one-dimensional beam model (**bending only**):

Mode	Natural Frequency ω_{Theory} [rad/s]
1	
2	
3	
4	

Table 2 C A S E 2 . Analyse the beam with fixed cross-section at z = 0 and pinned cross-section at z = L

Mode	frequency **f** $_{FEM}$ [Hz]	Natural freq. ω_{FEM} [rad/s]	Shape description
1			
2			
3			
4			
5			
6			
7			
8			

Table 3 C A S E 3 . Analyse the beam with fixed cross-sections.

Mode	frequency **f** $_{FEM}$ [Hz]	Natural freq. ω_{FEM} [rad/s]	Shape description
1			
2			
3			
4			
5			
6			
7			
8			

Table 3 C A S E 4 . Analyse the rotating cantilever beam *(ωy = 100 1/s)*.

Mode	frequency **f** $_{FEM}$ [Hz]	Natural freq. ω_{FEM} [rad/s]	Shape description
1			
2			
3			
4			
5			
6			
7			
8			

Conclusions:

➔ **Final report should include:**

- problem description
- short presentation of the FEM model (mesh, boundary conditions)
- table with obtained results (frequencies)
- graphs with distribution of normal stresses σz for the first 8 vibration modes
- discussion of results (comparison with simplified analytical solution)